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Quantum Creation of Highly Massive Particles in the Very Early Universe

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Quantum creation of very massive particles in the gravitational background of anisotropically perturbed Minkowski space-time is discussed. In this framework of semiclassical gravity the quantum mechanically produced heavy particles which made the initial space-time unstable and ushered into the FRW expansion phase at the Planck order epoch of the universe can account for the energy density at that epoch. Also, both the conformal and nonconformal particle-creations in the FRW era of the early universe after the Planck order epoch are investigated. In this consideration the total particle number of the observable universe as well as the present value of photon-to-baryon ratio are obtained in agreement with their accepted values from the observational facts. The existence of very massive particles at the very early period of the universe is also discussed here with the suggestion of an observational test.

1. INTRODUCTION

The quantum creation of matter in the very early universe has profound implications in its evolution. Several authors have considered the particle-creations for different fields in Friedmann space-time and also made calculations of the vacuum polarization (Birrell and Davies, 1982; Grib *et al.*, 1994 and references therein). The matter sources were treated in these works quantum mechanically in the framework of semi-classical gravity (Brout *et al.*, 1978, 1979a,b, 1980). In most of these articles, the gravitational backgrounds have only the modelling character without any correspondence to the actual cosmological scenario. On the other hand, in other cases of possible realistic situations the created particles cannot account for the actual energy density of the universe.

From such a quantum creation phenomenon one can have a nonsingular origin of the universe. It can even influence the geometry of the subsequence period and can also attend the problem related to the observed aspect of remarkable isotropy of the universe (Hartle and Hu, 1979; Hu and Parker, 1978; Lukash and Starobinsky,

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1974; Zel'dovich and Starobinsky, 1971). In fact, the initial anisotropy could have been dissipated by the production of particle pairs in the very early eras of the universe. In Nardone (1989) we find that the quantum production of very massive particles in the gravitational background of Minkowski space-time perturbed anisotropically can be the cause of the instability of that space-time ushering it in to an expansion phase. To make the Minkowski space-time unstable the masses of the produced particles should necessarily be more or around 50 times the Planck mass. Such massive particles might have been created quantum mechanically in the "zero epoch" of the universe and the produced matter can correspond to the desired energy density at the Planck order epoch time which marks the beginning of the expansion phase of the universe (De, 1993b). These particles of the very early era may be either primordial black holes or known elementary particles whose masses attain their present values at the present epoch of the universe because of the property of epoch-dependence of masses. That is, the particles like electrons, muons, massive neutrinos, etc., could have masses around 50 times the Planck mass at the Planck order epoch time of the universe. Presently, the quantum creation of particles after this Planck order epoch will be considered in the framework of semiclassical gravity. The matter field is taken here as the scalar field and is treated quantum mechanically.

We begin with the following section describing, in brief, the basic equations and the method of solution as applied to the previously considered case of matter-creation in the anisotropically perturbed Minkowski space-time. In the next Section 3, both the conformal and the nonconformal particle-creations after the Planck order time will be discussed. There we shall finally obtain the total particle number of the observable universe and also the present value of photon-to-baryon ratio. In the concluding Section 4, some remarks will be made about the particles of very large masses in the very early era of the universe. Also, a suggestion is given there on the possible observational test of their existence. In the following we shall use the unit $\hbar = c = k_B = 1$ and set $G = 1/8\pi m_{\mathrm{p}\ell}^2$, $t_{\mathrm{p}\ell}^2 = G$.

2. QUANTUM CREATION OF MATTER

We shall first obtain the field equations for the isotropic space-time which is the conformal Minkowski space-time related to FRW space (in four dimensions) with flat spatial sections. The mode decomposition for a scalar field ϕ in this space-time is given by

$$\phi(\mathbf{x}) = \int d^3k\,[a_k\,u_k(\mathbf{x}) + a_k^\dagger\,u_k^*(\mathbf{x})] \tag{1}$$

Here, the modes u_k can be written in the following separated form (Birrell and Davies, 1982):

$$u_k(\mathbf{x}) = (2\pi)^{-3/2}\, e^{i\vec{k}\cdot\vec{x}}\, \Omega(\eta)\chi_k(\eta) \tag{2}$$

where $\Omega(\eta)$ is the conformal factor and η is the conformal time parameter. Here, $k = |\vec{k}|$, and χ_k satisfies the following equation

$$\frac{d^2\chi_k}{d\eta^2} + \left[k^2 + \Omega^2(\eta)\left\{m^2 + \left(\xi - \frac{1}{6}\right)R(\eta)\right\}\right]\chi_k = 0 \tag{3}$$

with the normalization condition

$$\chi_k \partial_\eta \chi_k^* - \chi_k^* \partial_\eta \chi_k = i \tag{4}$$

The above equations are, in fact, derived from the following Lagrangian density

$$\mathscr{L}(x) = \frac{1}{2}[-g(\mathbf{x})]^{1/2}\{g^{\mu\nu}(\mathbf{x})\,\phi(\mathbf{x})_{,\mu}\,\phi_{,\nu} - [m^2 + \xi R(\mathbf{x})]\,\phi^2(\mathbf{x})\} \tag{5}$$

with the resulting action

$$S = \int \mathscr{L}(x)\, d^4x \tag{6}$$

where m is the mass of the scalar. Here, $R(\mathbf{x})$ represents the Ricci scalar and ξ is a numerical factor that represents the nature of the coupling between the scalar and the gravitational fields. The variation of the action S with respect to the scalar field ϕ is demanded by the action principle and this can give rise to the following field equation for the scalar field ϕ:

$$[\Box + m^2 + \xi R(\mathbf{x})]\,\phi(\mathbf{x}) = 0 \tag{7}$$

where

$$\Box\phi = (-g)^{-1/2}\partial_\mu[(-g)^{1/2}g^{\mu\nu}\partial_\nu\phi] \tag{8}$$

with $\eta = x^0$.

For the particularly interesting case of conformal coupling for which $\xi = 1/6$, we have the following equation for χ_k:

$$\frac{d^2\chi_k}{d\eta^2} + \omega_k^2(\eta)\chi_k = 0 \tag{9}$$

where

$$\omega_k^2(\eta) = k^2 + m^2\Omega^2(\eta) \tag{10}$$

In the following we shall consider an anisotropic perturbation (of extremely short duration) in the Minkowski space-time and find out the energy density of the matter created quantum mechanically with the very massive particles (of masses $\geq 50\ m_{\mathrm{p}\ell}$) around the Planck order time. Due to epoch-dependence of particle-masses, the masses of subatomic particles could have been of the order of more than 50 times the Planck mass around the Planck order epoch times. In fact, the mass of a particle could have been 53.3 times $m_{\mathrm{p}\ell}$ at the epoch time $\hat{t} = 0.05\ t_{\mathrm{p}\ell}$. The result is obtained from the mass relation (De, 1991, 1997)

$$m = \bar{m}(1 + 2\alpha H(t)) \tag{11}$$

where $\alpha = 0.26 \times 10^{-23}$ s and $\bar{m}$ is the "inherent" mass of the particle. It corresponds to the present mass of the particle with an extremely high degree of accuracy. Here, in fact, the Hubble parameter is given by $H(t) = (2/3t)$ for the FRW matter-dominated expansion phase (De, 1993a), which begins at the Planck order time due to the instability caused by these heavy massive particles, as shown by Nardone (1989). The conformal factor $\Omega(\eta)$ for this period of matter-dominated FRW expansion phase after the Planck order time is given by

$$\Omega(\eta) = A^3\left(\frac{\eta}{\tau}\right)^2 \tag{12}$$

where A is an appropriate constant and $\tau = 2H_0^{-1}$, H_0 being the present value of H. On the other hand, the space-time before the Planck order time $\hat{t}$ was an anisotropically perturbed Minkowski space-time. The line element of this space-time is given by

$$ds^2 = \Omega^2(\xi)\left[d\xi^2 - \sum_{i=1}^{3}\{1 + h_i(\xi)\}\,(dx^i)^2\right] \tag{13}$$

where

$$\Omega(\xi) = B^3\left(\frac{\hat{\xi}}{\tau}\right)^2$$

$\hat{\xi}$ being the conformal time parameter corresponding to $\hat{t}$. From the continuity of the conformal factors (and also of the corresponding scale factors) we have the following relations

$$3^{2/3}A = B \qquad \hat{\eta} = 3\hat{\xi} \tag{14}$$

where $\hat{\eta}$ corresponds to $\hat{t}$ also. The epoch time and the conformal time parameters are related by

$$A\eta = (3\tau^2 t)^{1/3} \quad \text{and} \quad \xi = \frac{t}{B}\left(\frac{\tau}{\hat{t}}\right)^{2/3} = \frac{t}{A}\left(\frac{\tau}{3\hat{t}}\right)^{2/3} \tag{15}$$

The perturbation functions $h_i(\xi)$ $(i = 1, 2, 3)$ in (13) are specified as follows:

$$h_i(\xi) = e^{-\alpha\xi^2}\ \cos(\beta\xi^2 + \delta_i) \tag{16}$$

where the constants α, β, and δ_i are related to the duration of the perturbation. For a scalar field $\phi(\boldsymbol{x})$ in this anisotropically perturbed Minkowski space-time the mode decomposition and the separated form of modes given by (1) and (2) remain the same but the equation for χ_k is to be modified to (retaining only the first order terms in h_i)

$$\frac{d^2\chi_k}{d\xi^2} + \left\{k^2 + m^2\Omega^2(\xi) - \sum_{i=1}^{3} h_i(\xi)\, k_i^2\right\}\chi_k = 0 \tag{17}$$

We follow the method of solution for this equation as given in Birrell and Davies (1980) who developed an original method of Zel'dovich and Starobinsky (1971, 1977) for the case of small perturbations about a FRW space-time. Apart from taking h_i to be very small compared to unity, one can impose, for simplicity, the condition

$$\sum_{i=1}^{3} h_i(\xi) = 0 \tag{18}$$

which gives rise to the fact that δ_i must differ from one another by $2\pi/3$. It is evident that the following conditions of the present method of approximation hold good:

$$\begin{aligned} &h_i(\xi) \to 0 \quad \text{as } \xi \to \pm\infty \\ &\Omega^2(\xi) \to \Omega^2(\infty) = \Omega^2(-\infty) < \infty \quad \text{as } \xi \to \pm\infty \end{aligned} \tag{19}$$

Then the normalized positive frequency solution of (17) as $\xi \to -\infty$ is given by

$$\chi_k^{\text{in}}(\xi) = (2\omega)^{-1/2}\, e^{-i\omega\xi} \tag{20}$$

where

$$\omega^2 = k^2 + m^2\Omega^2(\pm\infty) \tag{21}$$

Now, with the initial condition the Eq. (17) can be transformed into the following integral equation:

$$\chi_k(\xi) = \chi_k^{\text{in}}(\xi) + \omega^{-1}\int_{-\infty}^{\xi} V_k(\xi')\,\sin[\omega(\xi - \xi')]\chi_k(\xi')\, d\xi' \tag{22}$$

where

$$V_k(\xi) = \sum_{i=1}^{3} h_i(\xi)\, k_i^2 \tag{23}$$

The Eq. (22) possesses the following solution in the late time region:

$$\chi_k^{\text{out}}(\xi) = \alpha_k\chi_k^{\text{in}}(\xi) + \beta_k\chi_k^{\text{in}*}(\xi) \tag{24}$$

where α_k and β_k are the Bogolubov coefficients. For this case these are given by

$$\alpha_k = 1 + i \int_{-\infty}^{\infty} \chi_k^{\text{in}*}(\xi) V_k(\xi) \chi_k(\xi)\, d\xi \tag{25}$$

$$\beta_k = -i \int_{-\infty}^{\infty} \chi_k^{\text{in}}(\xi) V_k(\xi) \chi_k(\xi)\, d\xi \tag{26}$$

For small $V_k(\xi)$ we can solve (22) by iteration. The Bogolubov coefficients to the first order in $V_k(\xi)$ are found to be

$$\alpha_k = 1 + \frac{i}{2\omega} \int_{-\infty}^{\infty} V_k(\xi)\, d\xi \tag{27}$$

$$\beta_k = -\frac{i}{2\omega} \int_{-\infty}^{\infty} e^{-2i\omega\xi} V_k(\xi)\, d\xi \tag{28}$$

since $\chi_k(\xi) = \chi_k^{\text{in}}(\xi)$ to the lowest order in $V_k(\xi)$. The energy density per unit proper volume, which is related to β_k is given by

$$\rho = \frac{1}{(2\pi)^3 \Omega^4} \int |\beta_k|^2 \omega\, d^3k \tag{29}$$

It is evident from the expression of $V_k(\xi)$ in (23) that only the anisotropy of the space-time contributes to the energy density. The energy density is, in fact, given as (Birrell and Davies, 1982)

$$\rho = \frac{\tilde{m}^2}{1536\, \pi^{1/2} \Omega^4} \frac{(\alpha^2 + \beta^2)^{3/2}}{\alpha^2} \exp\left\{-3\alpha \frac{\tilde{m}^2}{\alpha^2 + \beta^2}\right\} W_{-3/2,3/2}\left(\frac{2\alpha \tilde{m}^2}{\alpha^2 + \beta^2}\right) \tag{30}$$

where W is a Whittaker function and

$$\tilde{m} = \Omega(\pm\infty) m = B^3(\hat{\xi}/\tau)^2 m = A(3\hat{t}/\tau)^{2/3} m \tag{31}$$

Here m is the mass of the particle at the Planck order epoch time $\hat{t}$ and can be obtained from the mass relation (11). By taking m as the mass of the muon, a representative particle, at $\hat{t}$ and using an integral representation of the Whittaker function it is possible to find an expression for the energy density as (De, 1993b)

$$\rho = 2.06 \times 10^3 (1 + p^2)^{1/2} a^5 m_{\text{p}\ell}^4\, e^{-2/a^2} \int_0^{\infty} \frac{e^{-x} x^{5/2}}{\sqrt{1 + a^2 x}}\, dx \tag{32}$$

where we have set

$$\alpha = A^2 m_{\text{p}\ell}^2 \delta^2 \quad \beta = p\alpha \quad \delta^2 = 7.8 \times 10^{-80} \{a^2/(1 + p^2)\} \tag{33}$$

It is to be noted that the constants a and p determine the "duration" and "frequency" of the anisotropic fluctuations. With possible choices for them one can find the energy density numerically. For one such choice, $a = 2$, $p = 1$, the energy density at $\hat{t}$ is given by

$$\rho = 2.8 \times 10^4 m_{\mathrm{p}\ell}^4 \tag{34}$$

After the epoch $\hat{t}$ the expansion phase of the universe began due to the instability caused by the very massive particles. In fact, Nardone (1989) has shown that instability of Minkowski space-time corresponding to a global fluctuation $\delta(t)$, that is, for the scale factor $a(t)$ given by

$$a(t) = 1 + \delta(t), \tag{35}$$

shows up as soon as

$$Km^2 \geq 288\,\pi^2 \quad (K = 8\pi G) \quad \text{or } m \geq 53.3\, m_{\mathrm{p}\ell} \tag{36}$$

The expansion phase after the epoch $\hat{t}$ was the matter-dominated FRW universe which continued upto the transition epoch $\alpha = 0.26 \times 10^{-23}$s, when the particles became relativistic and contributed to the radiation energy density. In fact, after this epoch the radiation era with standard cosmology set in. The energy density is then given by

$$\rho = \left(\frac{\hat{t}}{t}\right)^2 \rho(\hat{t}) \tag{37}$$

and this density at the transition epoch α is fully contributed from the radiation energy density at that epoch. Thus, it can be calculated and by using the standard relation one can also find the universe temperature at the epoch α. These are given by

$$\rho(\alpha) = 6.14 \times 10^{90}\ \mathrm{cm}^{-4} \quad T(\alpha) = 5.52 \times 10^{22}\ \mathrm{cm}^{-1} \tag{38}$$

These results are in good agreement with the results that are calculated from the standard cosmology.

Now, the perturbation functions $h_i(\xi)$ can be found for the specific values of the parameters a and p given above. These are given by

$$h_i(\xi) = e^{-(t/0.9\hat{t})^2} \cos\left\{\left(\frac{t}{0.9\hat{t}}\right)^2 + \delta_i\right\} \tag{39}$$

The amplitude of the perturbation functions are damped for $t \gg 0.9\hat{t}$. That is, the anisotropic fluctuation of the Minkowski space-time died out after the epoch $\hat{t}$. The fluctuation was dominant for $|t| < 0.9\hat{t}$. This dominant "duration" of the "fluctuation era" depends on the parameter-values which also determine the energy density. Thus, the duration is related to the energy density. Thus, the duration of the anisotropically perturbed Minkowski space-time for such a short period, as described above, could produce particles with masses 55.5 $m_{\mathrm{p}\ell}$ (evident from the mass relation (11)) that contributed to the matter energy density at $\hat{t}$. Again, this energy density at $\hat{t}$ determines the radiation energy density at the transition epoch α (and consequently, at the late time of the universe) correctly. It is here seen that the "zero" epoch (the cosmological time-origin) of the present universe, given by $\xi = 0 = t$, corresponds to the maximum amplitudes of the perturbation functions and the universe is free from initial singularity.

3. PARTICLE CREATION AFTER PLANCK ORDER EPOCH

In De (1993a) the era after the Planck order epoch time $\hat{t}$ is a matter-dominated FRW expansion stage of the universe. The conformal Minkowski space-time related to this era has the conformal factor given in (12). The corresponding scale factor of FRW universe is

$$R(t) = A\left(\frac{3t}{\tau}\right)^{2/3} \tag{40}$$

For the period, $\hat{t} \leq t \lesssim \alpha$, we have from (11)

$$m \simeq 2\alpha\bar{m}H(t) = \frac{4\alpha\bar{m}\,\tau^2}{A^3\eta^3}$$

Therefore, we get

$$m\Omega(\eta) = \frac{4\alpha\bar{m}}{\eta} \quad \text{for } \hat{\eta} \leq \eta \lesssim \eta_\alpha$$

where η_α is the conformal time parameter corresponding to the epoch time α. The Eq. (9) becomes

$$\left.\begin{aligned} &\frac{d^2\chi_k}{d\eta^2} + \omega_k^2(\eta)\chi_k = 0 \\ &\omega_k^2(\eta) = k^2 + \frac{16\alpha^2\bar{m}^2}{\eta^2} \quad \text{for } \eta \geq \hat{\eta} \end{aligned}\right\} \tag{41}$$

with the normalization condition for χ_k given by (4). For nonconformal particle-creation, that is, for the quantized scalar field with arbitrary coupling in a nonstationary isotropic gravitational field, the equation for χ_k follows from (3) which can be written as follows:

$$\frac{d^2\chi_k}{d\eta^2} + Q^2(\eta)\chi_k = 0 \tag{42}$$

where

$$\left.\begin{aligned} Q^2(\eta) &= \omega_k^2(\eta) + q(\eta) \quad \text{for } \eta \geq \hat{\eta} \\ q(\eta) &= (6\xi - 1)\frac{\Omega''(\eta)}{\Omega(\eta)} \end{aligned}\right\} \tag{43}$$

For both the cases the normalized positive frequency solution for χ_k as $\eta \to -\infty$ is given by

$$\chi_k^{\text{in}}(\eta) = (2\omega)^{-1/2}\, e^{-i\omega\eta} \tag{44}$$

where

$$\omega^2 = k^2 + \frac{16\alpha^2\bar{m}^2}{\hat{\eta}^2}$$

In fact, the space-time before the conformal time $\hat{\eta}$ was Minkowskian with a constant conformal factor given in the previous section. Thus, $q(\eta) = 0$ and $Q^2(\eta) = \omega_k^2(\eta) \equiv \omega^2$. The Eqs. (41) and (42) for the conformal and nonconformal cases respectively can be written as the following integral equation:

$$\chi_k(\eta) = \chi_k^{\text{in}}(\eta) + \frac{1}{\omega}\int_{\hat{\eta}}^{\eta} V(\xi)\chi_k(\xi)\, \sin\{\omega(\eta - \xi)\}\, d\xi \tag{45}$$

Here, $V(\eta)$ for the two cases are respectively

$$V(\eta) = m^2\{\Omega^2(\hat{\eta}) - \Omega^2(\eta)\} = 16\alpha^2\bar{m}^2\left(\frac{1}{\hat{\eta}^2} - \frac{1}{\eta^2}\right) \tag{46}$$

and

$$V(\eta) = 16\alpha^2\bar{m}^2\left(\frac{1}{\hat{\eta}^2} - \frac{1}{\eta^2}\right) - 12\left(\xi - \frac{1}{6}\right)\frac{1}{\eta^2} \tag{47}$$

In the late time region the Eq. (45) possesses the solution (Birrell and Davies, 1982).

$$\chi_k^{\text{out}}(\eta) = \alpha_k\chi_k^{\text{in}}(\eta) + \beta_k\chi_k^{\text{in}*}(\eta) \tag{48}$$

where the Bogolubov coefficients α_k and β_k are given by

$$\left.\begin{aligned} \alpha_k &= 1 + i\int_{\hat{\eta}}^{\infty} \chi_k^{\text{in}*}(\eta)V(\eta)\chi_k(\eta)\, d\eta \\ \beta_k &= -i\int_{\hat{\eta}}^{\infty} \chi_k^{\text{in}}(\eta)V(\eta)\chi_k(\eta)\, d\eta \end{aligned}\right\} \tag{49}$$

From (48) we can find, by using (44)

$$\frac{i}{\omega}\,\frac{d\chi_k^{\text{out}}(\eta)}{d\eta} = \alpha_k\frac{e^{-i\omega n}}{(2\omega)^{1/2}} - \beta_k\frac{e^{+i\omega n}}{(2\omega)^{1/2}}$$

Consequently, it follows that

$$\chi_k^{\text{out}}(\eta) - \frac{i}{\omega}\,\frac{d\chi_k^{\text{out}}(\eta)}{d\eta} = 2\beta_k\frac{e^{+i\omega n}}{(2\omega)^{1/2}} \tag{50}$$

Therefore, we get

$$
\begin{aligned}
\frac{2}{\omega}|\beta_k|^2 &= \left|\chi_k^{\text{out}}(\eta)\right|^2 + \frac{1}{\omega^2}\left|\frac{d\chi_k^{\text{out}}(\eta)}{d\eta}\right|^2 \\
&\quad + \frac{i}{\omega}\left\{\chi_k^{\text{out}}(\eta)\frac{d\chi^{\text{out}*}(\eta)}{d\eta} - \chi_k^{\text{out}*}(\eta)\frac{d\chi_k^{\text{out}}(\eta)}{d\eta}\right\} \\
&= \left|\chi_k^{\text{out}}(\eta)\right|^2 + \frac{1}{\omega^2}\left|\frac{d\chi_k^{\text{out}}(\eta)}{d\eta}\right|^2 + \frac{i^2}{\omega}
\end{aligned}
$$

[because of the normalization condition (4)]

Thus, we arrive at the following expression for $|\beta_k|^2$:

$$
|\beta_k|^2 = \frac{1}{2\omega}\left\{\left|\frac{d\chi_k^{\text{out}}(\eta)}{d\eta}\right|^2 + \omega^2\left|\chi_k^{\text{out}}(\eta)\right|^2\right\} - \frac{1}{2} \tag{51}
$$

This formula may be compared with the spectral quasiparticles density, related to unit volume, given by the squared modulus of the Bogoliubov transformation coefficient in the diagonalization procedure of the quantized field Hamiltonian in creation-annihilation operators (Grib *et al.*, 1994).

Now, the correctly normalized exact solutions of the Eqs. (41) and (42) for χ_k can be written as

$$
\chi_k(\eta) = \frac{1}{2}\, e^{-i\nu\pi/2}\, (\pi\eta)^{1/2} H_\nu^{(2)}(k\eta) \tag{52}
$$

where

$$
\nu^2 = \frac{1}{4} - 16\alpha^2\bar{m}^2 \tag{53a}
$$

and

$$
\nu^2 = \frac{1}{4} - 16\alpha^2\bar{m}^2 - 12\left(\xi - \frac{1}{6}\right) \tag{53b}
$$

respectively for the conformal and nonconformal cases. Here, $H_\nu^{(2)}$ is a Hankel function of the second kind. That this solution (52) is normalized according to (4) can be verified by using the following formula

$$
H_\nu^{(2)}(x)\,\frac{d}{dx}H_\nu^{(2)*}(x) - H_\nu^{(2)*}(x)\frac{d}{dx}H_\nu^{(2)}(x) = \frac{4i\,e^{i\nu\pi}}{\pi x}
$$

where $\nu = ib$ is purely imaginary. This formula, in fact, can be deduced from the Wronskian of the pair of Hankel functions $H_\nu^{(1)}(x)$ and $H_\nu^{(2)}(x)$, which is equal to $(-4i/\pi x)$ and by using the following integral representations of $H_\nu^{(1)}(x)$

and $H_\nu^{(2)}(x)$:

$$H_\nu^{(1)}(x) = \frac{2\,e^{-\nu\pi i/2}}{\pi i}\int_0^\infty e^{ixcht} ch\;\nu t\;dt \tag{54a}$$

$$H_\nu^{(2)}(x) = -\frac{2\,e^{-\nu\pi i/2}}{\pi i}\int_0^\infty e^{-ixcht} ch\;\nu t\;dt$$

$$\text{for } -1 < \text{Re}\,\nu < 1,\; x > 0 \tag{54b}$$

(Gradshteyn and Ryzhik, 1980). It is to be noted that the solution (52) is consistent with the condition to be satisfied in order to have an adiabatic vacuum a reasonable definition of a no-particle state as $\eta \to \pm\infty$. This condition is, in fact, that the Ath order adiabatic approximation $\chi_k^{(A)}$ to χ_k should satisfy

$$\chi_k^{(A)} \xrightarrow[k,\eta\to\infty]{} \frac{1}{(2k)^{1/2}}\,e^{-ik\eta} \tag{55}$$

for large k or η (for details see Birrell and Davies, 1982). It is pointed out above that $\nu = ib$ is purely imaginary. In fact, by using the present mass of a muon (the representative particle) we find $\alpha\bar{m} = 0.416$ (in the unit $c = \hbar = 1$), and consequently from (53a,b) it follows that

$$\nu^2 = -2.519 \tag{56a}$$

and

$$\nu^2 = -0.519 - 12\xi < 0 \quad \text{for } \xi \geq 0 \tag{56b}$$

for conformal and nonconformal cases respectively.

From the solution (52) for $\chi_k(\eta)$ $(\eta \geq \hat{\eta} > 0)$ one can find $\chi_k^{\text{out}}(\eta)$. This is given as

$$\chi_k(\eta) \xrightarrow[\eta\to\infty]{} \chi_k^{\text{out}}(\eta) \tag{57}$$

by using an asympotic formula for the Hankel function $H_\nu^{(2)}$, which is

$$H_\nu^{(2)}(k\eta) \sim \left(\frac{2}{\pi k\eta}\right)^{1/2} \exp\left\{-i\left(k\eta - \frac{\nu\pi}{2} - \frac{\pi}{4}\right)\right\} \quad \text{for large } |k\eta| \tag{58}$$

With this solution for $\chi_k^{\text{out}}(\eta)$ we can find out from (51) the following expression of $|\beta_k|^2$ for the conformal case:

$$|\beta_k|^2 = \frac{(\omega - k)^2}{4\omega k} \tag{59}$$

It is interesting to note that $|\beta_k|^2$ is independent of the value of ν or b and therefore, for nonconformal case we have the same expression (59) for $|\beta_k|^2$. Also, we see that $|\beta_k|^2$ vanishes as k becomes large because $\omega \to k$ for large k.

Now, the number of particles created in the Lagrange volume by the gravitational field until the moment η is

$$N(\eta) = \Omega^3(\eta)n(\eta) \tag{60}$$

where the number density $n(\eta)$ is given by

$$n(\eta) = \frac{1}{2\pi^2\Omega^3(\eta)}\int_0^\infty k^2\, dk\, |\beta_k|^2 \tag{61}$$

Here, the conformal factor $\Omega(\eta) \equiv R(t)$ is given by (12) and (40). We can now calculate the total particle number N for large η.

$$N = \frac{1}{2\pi^2}\int_0^\infty k^2\, dk\, |\beta_k|^2 \quad \text{for large } \eta \tag{62}$$

For large η, $|\beta_k|^2$ is given by (59) and consequently we have

$$N = \frac{1}{2\pi^2}\int_0^\infty \frac{k}{4\omega}(\omega - k)^2\, dk = \frac{8}{3\pi^2}\left(\frac{\alpha\bar{m}}{\hat{\eta}}\right)^3 \tag{63}$$

From (15), it follows that

$$A\hat{\eta} = (3\tau^2\hat{t})^{1/3} \quad \text{and} \quad A\eta_\alpha = (3\tau^2\alpha)^{1/3} \tag{64}$$

Consequently,

$$\frac{\hat{\eta}}{\eta_\alpha} = \left(\frac{\hat{t}}{\alpha}\right)^{1/3} = 10^{-7} \quad (\text{since } \hat{t} = 0.05\, t_{\mathrm{p}\ell}) \tag{65}$$

Also,

$$R(\alpha) = \Omega(\eta_\alpha) = A^3\left(\frac{\eta_\alpha}{\tau}\right)^2 = \frac{3\alpha}{\eta_\alpha} = \frac{3\alpha}{\hat{\eta}}\left(\frac{\hat{t}}{\alpha}\right)^{1/3} \quad (\text{using (64 and (65))})$$

Therefore,

$$\frac{1}{\hat{\eta}^3} = R^3(\alpha)\left(\frac{\alpha}{\hat{t}}\right) = \frac{1}{27\alpha^3}$$

and we have, from (63), the total particle number N as

$$N = \frac{8\alpha\bar{m}^3}{81\pi^2\hat{t}}R^3(\alpha) \tag{66}$$

As discussed in De (1993a) the expansion phase of the very early universe after the epoch time $\hat{t}$ changed to a radiation-dominated FRW expansion stage with standard cosmology at the epoch α. In De (1999), the early universe was considered in the framework of modified general relativity originally proposed by Rastall (1972) and later by Al-Rawaf and Taha (1996a,b). There it was shown that the evolution in the era $(\hat{t}, \alpha)$ is a mild inflation with the scale factor proportional to the epoch time. This mild inflation was also shown to have turned off automatically and transited into the radiation era with the usual cosmology after the epoch time α. In this connection it is to be noted that the expression for $m\Omega(\eta)$ given above is valid upto the epoch time α which correponds to the conformal time parameter η_α. Now, it will be shown that after the epoch time α, when the universe is radiation-dominated FRW universe with the conformal factor

$$\Omega(\eta) = b\eta \tag{67}$$

corresponding to the scale factor $R(t) = (2bt)^{1/2}$, there is no quantum creation of particles by the gravitational field. For this purpose it is convenient to employ the Hamiltonian diagonalization procedure. If we consider the conformal particle-creation then the formula for $|\beta_k|^2$ is given by

$$|\beta_k|^2 = \frac{1}{2\omega_k(\eta)}\left(|\partial_\eta \chi_k|^2 + \omega_k^2(\eta)|\chi_k|^2\right) - \frac{1}{2} \tag{68}$$

On the other hand, if nonconformal particle production is considered then the same formula (68) for the spectral quasiparticle density given by $|\beta_k|^2$ remains valid with a "changed" concept of particles in this nonconformal case (Bezerra *et al.*, 1997). There, in fact, $|\beta_k|^2$ is, by definition, equated to R.H.S. of (68) with the "switched off" external field in the general expression for it in the case $\xi \neq 1/6$ and, of course, with the replacement of $\omega_k(\eta)$ by $Q(\eta)$. Thus, for nonconformal case

$$|\beta_k|^2 = \frac{1}{2Q(\eta)}(|\partial_\mu \chi_k|^2 + Q^2(\eta)\,|\chi_k|^2) - \frac{1}{2} \tag{69}$$

For the conformal factor (67), we have $q(\eta) = 0$ and hence

$$Q^2(\eta) = \omega_k^2(\eta) = k^2 + \bar{m}^2\left(b\eta + \frac{2\alpha}{\eta}\right)^2 \quad \text{(by using the mass relation (11)).}$$

Consequently, for $\eta > \eta_\alpha$ we have

$$Q^2(\eta) = \omega_k^2(\eta) \simeq k^2 + \bar{m}^2 b^2 \eta^2 \tag{70}$$

(In fact, at $\eta = \eta_\alpha$, $b\eta_\alpha = 2\alpha/\eta_\alpha$ and as $\eta > \eta_\alpha$, $b\eta > 2\alpha/\eta$).

Thus, we get the same equation for χ_k for both the conformal and nonconformal cases. Also, the expressions for $|\beta_k|^2$ are the same for these cases. Now, in Birrell and Davies (1982) the exact solution for χ_k for the case with $\omega_k(\eta)$ given by (70) has been given. For large η, $\chi_k(\eta)$ has been found to be

$$\chi_k(\eta) = (2\bar{m}b|\eta|)^{-1/2}\, e^{-i\bar{m}b\eta^2/2} \tag{71}$$

($\eta > 0$, therefore $|\eta| = \eta$, η large)

$$\partial_\eta \chi(\eta) = \frac{1}{(2\bar{m}b|\eta|)^{1/2}}\left\{i\bar{m}b\eta + \frac{1}{2\eta}\right\} e^{-i\bar{m}b\eta^2/2}$$

Therefore, for large η,

$$|\beta_k|^2 = \frac{1}{16(\bar{m}b)^2\eta^4} \to 0$$

That is, there is no massive particle-creation in this radiation era after the conformal time parameter η_α or equivalently after the epoch time α. Thus, the particle-creation occurs only in $(\hat{t}, \alpha)$. In the consideration of particle-creation for the period $(\hat{t}, \alpha)$, $\chi_k(\eta)$ for large η has been regarded as $\chi_k^{\text{out}}(\eta)$. In fact, around the conformal time parameter η_α (that is, around the cosmological time α), the asymptotic formula for the Hankel function remains valid because of the fact that

$$k\eta_\alpha = k\hat{\eta}\left(\frac{\eta_\alpha}{\hat{\eta}}\right) \simeq \bar{m}\alpha\left(\frac{\eta_\alpha}{\hat{\eta}}\right) = 0\ (10^7)$$

Now, the scale factor in (66) can be obtained by using the standard cosmological invariant

$$RT = R_0T_0 = 1.18 \times 10^{29}\ u, \quad 1 < u < 1.8 \tag{72}$$

where R_0 and T_0 are the present scale factor and the universe temperature, respectively. They are given by

$$R_0 = 10^{28}u \text{ cm}, \quad T_0 = 11.8 \text{ cm}^{-1} \tag{73}$$

The cosmological invariant (72) is valid after the epoch time α, since thereafter the standard cosmology follows.

Now, from (38) and (72), we can find the scale factor at the epoch α. It is given by

$$R(\alpha) = 2.14 \times 10^6 u \text{ cm} \tag{74}$$

Consequently, from (66) we find the total particle number N of the created particles from the gravitational field. It is given by (using (65) and value of $\bar{\alpha}m$ given in the previous section)

$$1.5 \times 10^{76} < N < 8.6 \times 10^{76} \tag{75}$$

This particle number at the epoch α corresponds to the total particle number at the present epoch of the universe as it remains constant afterwards. This number is in agreement with the accepted value of the baryon number of the present universe (Kolb and Turner, 1990). Also one can compute the photon-to-baryon ratio at the epoch α from the radiation energy density and the universe temperature at that epoch, as given in (38). From the standard relation

$$n_\gamma = \frac{\pi}{13}T^3 \tag{76}$$

the photon number density n_γ at the epoch α is found to be

$$n_\gamma(\alpha) = 4.08 \times 10^{67} \text{ cm}^{-3} \tag{77}$$

On the other hand, the particle number density at the epoch α is obtained from (74) and (75). It is given by

$$n_m(\alpha) = 1.53 \times 10^{57}\ \mathrm{cm}^{-3} \tag{78}$$

Consequently, we have at the epoch time α

$$\frac{n_\gamma}{n_m} = 2.66 \times 10^{10} \tag{79}$$

As standard cosmology follows after the epoch α, this ratio remains constant afterwards and gives its present value.

4. CONCLUDING REMARKS: PARTICLES WITH VERY LARGE MASS

We have discussed the creation of heavy particles in the classical nonstationary space-times, that is, the anisotropically perturbed Minkowski, and FRW space-times. The quantum creation of particles from vacuum in the gravitational field is regarded as the "creation" of the universe at the Planck order epoch after which the matter-dominated "creation era" and subsequently (after a transition epoch) the usual radiation-dominated era follows. We have found that the created particles can give the total particle number and the photon-to-baryon ratio of the present universe in agreement with the accepted values of them. In an article (De, 2001) we have also found the specific entropy of the present universe from a phenomenological consideration of the very early universe with bulk viscosity in the framework of full casual thermodynamics and it is in good agreement with the accepted value.

The created particles with very large mass may either be known elementary particles such as muons, electrons, and massive neutrinos or the primordial black holes. The masses (of the order of Planck mass) of the elementary particles reduce to their present values at the present epoch of the universe owing to their epoch-dependence. Grib (1989) has discussed the quantum effects of vacuum polarization in early FRW space-time, which give rise to an effective change of the gravitational constant. Such a change in gravitational constant leads to the possibility of creation of particles with macroscopic masses to the order of the mass of the observable universe or some effective mass equivalent to its entropy. Subsequent change of the gravitational constant compels these particles to explode as black holes and his assertion is that only particles with microscopic masses can be created from the vacuum in the present era because of the present value of the gravitational constant. Thus, no big bang is possible now. Also, Schrödinger cats, observers and other macroscopic bodies cannot be created from the vacuum quantum mechanically at the present era. Only in the quantum era of the universe, when the gravitational constant is small enough, Schrödinger cats may be observable. With the change of the value of the gravitational constant G, the universe becomes macroscopic and classical. In Grib *et al.* (1994) the quantum creation of massive particles in strong field has also been discussed. In there, finite expressions for the density of created particles were obtained with the use of the method of diagonalization of the instantaneous Hamiltonian. The created particles are real and not virtual as the gravitational field acts as the energetic reservoir.

The interesting fact is that Grib (1989) obtained the following relation between the mass m of the created particles and the effective gravitational constant $\tilde{G}$:

$$\frac{1}{8\pi\tilde{G}} = \frac{1}{8\pi G} + \frac{m^2}{288\pi^2} \tag{80}$$

where G is the modern value of this constant. It is apparent that small value of $\tilde{G}$ makes the mass m of the created particles large. It is also clear from the uncertainty relation $\Delta E \Delta t \gtrsim h$ that the large mass can be created from vacuum if Δt is much less than the "modern" value of Planck time. In fact, for small $\tilde{G}$ one can have small Planck time $\tilde{t}_{\mathrm{p}\ell} \sim \tilde{G}^{1/2}$. Thus, it is possible for gravity to remain classical even for $t \ll t_{\mathrm{p}\ell} \sim G^{1/2}$. This fact justifies the present consideration of quantum creation of particles of Planck order masses from classical gravity around the Planck order time. Even the creation of heavier particles is possible at an earlier epoch of the universe from the classical gravity. From (79) and the relation for epoch-dependence of mass given in (11) which is, for $t \ll \alpha$,

$$m = 2\alpha\bar{m}H(t) = 0.832H(t) \tag{81}$$

we get

$$\frac{1}{8\pi\tilde{G}} = \frac{1}{8\pi G} + \frac{(0.832)^2(H(t))^2}{288\pi^2}$$

Since $H(t) = 2/3t$ for the matter-dominated FRW era of "creation" we find

$$\frac{1}{8\pi\tilde{G}} \simeq 2\left(\frac{0.832}{36\pi t}\right)^2 \quad \text{or } \frac{1}{4\tilde{t}_{\mathrm{p}\ell}} \simeq \frac{0.832}{36\sqrt{\pi}t} \quad \text{or } t \simeq 0.05\,\tilde{t}_{\mathrm{p}\ell} \tag{82}$$

Thus, we see that the epoch time of the creation of particles with large masses is of the order of the "effective," that is, the "changed" Planck time when the gravity can be regarded as classical. This is because of the change in G due to vacuum polarization effect. Thus, whatever large may be the mass of the created particles the gravity might be considered as classical at the creation epoch of these particles. These particles, as pointed out above, may be primordial black holes or even the known elementary particles such as muons, neutrinos, etc.

The epoch-dependence of particle-mass seems to be very encouraging for the question raised by Dicke and Peebles (1979) in respect of the beginning of the universe as a "quantum fluctuation" with $K^{-1} \sim 1$ where $K = Gm_{\mathrm{p}}^2/\hbar c$ (m_{p} being the mass of the proton). In fact, in our framework $K^{-1} \sim 1$ (in the unit $\hbar = c = k_B = 1$, $Gm_{\mathrm{p}\ell}^2 \sim 1$ and since at the Planck time $t_{\mathrm{p}\ell}$ the proton mass $m_{\mathrm{p}} \sim m_{\mathrm{p}\ell}$). The generation of the present large value of K^{-1} is possible because the mass of the proton decreases from its Planck-scale value to its modern value at the present epoch. It is also mentioned here that the GUT and SUSY theories require particles with such large masses (neutrinos) at the early universe. The epoch-dependence of mass admits all such Planck-scale massive particles.

Parker (1989) pointed out the possible existence of particles with masses of the order of Planck mass ($0.28\ m_{\mathrm{p}\ell}$) in the very early universe (at the Planck time $t_{\mathrm{p}\ell}$) in his discussion of possible anomalous decay of the neutral pion into gravitons. There it is supposed that a massive neutral particle should appear as an interpolating field in the divergence of an axial current, which contains gravitational and electromagnetic anomalies. If the mass of the decaying particle is of Planck-order ($0.28\ m_{\mathrm{p}\ell}$) then the gravitational decay rate Γ (that is, for the decay of π^0 at rest into a pair of gravitons) becomes the same order in magnitude as the electromagnetic decay rate Γ_{em} of π^0 (that is, for $\pi^0 \to 2\gamma$). In fact, the ratio of these decay rates is given by

$$\frac{\Gamma}{\Gamma_{\mathrm{em}}} = \frac{1}{36\pi\alpha^2}\left(\frac{m}{m_{\mathrm{p}\ell}}\right)^4 \tag{83}$$

Here, α is the fine structure constant. Now, if π^0 goes through a state of interpolating π^0 field which is regarded as a "cluster" of muon–antimuon or neutrino–antineutrino pair having epoch-dependent masses then we can have very large mass $m = 0.555 m_{\mathrm{p}\ell}$ at the Planck time $t_{\mathrm{p}\ell}$. Thus, the interpolating cluster field (a neutral particle field) having such a very large mass in the very early period of the universe gives rise to a significant decay rate for π^0 decaying into a pair of gravitons. This gravitational decay of π^0 via the anomaly, as discussed by Parker (1989), should result in a nonthermal cosmic gravitational wave background at a frequency characteristic of the rest energy of the decaying particle. It is expected that in the future progress of gravitational wave detection one can have a possible observational test for the large mass of the particle due to its epoch-dependence by studying the nature of the resulting gravitational wave from such decays, if they exist, in the early universe.

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